

AD/RHIC-AP-47

Analysis of the Decapole Systematic Error
in the Dipoles and of the Correctors

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(1)

The equations of motion in the presence of a decapole term are

$$x'' + K_H x = \frac{b_4}{\rho} (x^4 - 6x^2y^2 + y^4) \quad (1a)$$

$$y'' + K_V y = -4 \frac{b_4}{\rho} (x^3y - xy^3) \quad (1b)$$

where ρ is the bending radius in the dipoles and b_4 is the strength of the decapole term.

We can calculate the first-order contribution to the tune-shift vs. betatron angle-difference and off-momentum value, by separating the free betatron oscillations from the closed-orbit deviation

$$\tilde{x} = \eta \delta + \hat{x}, \quad \tilde{y} = \hat{y}$$

\hat{x}, \hat{y} free betatron oscillations

η , dispersion (only in the horizontal plane)

δ , off-momentum value

~~Effect of decapole term in y^4 gives the N-cusp effect since it does not give any first-order contribution to tune-shift. Then subtracting from (1a and b) the equations for the closed orbit~~

$$\begin{aligned}\ddot{\tilde{x}} + k_x \tilde{x} &= \frac{b_4}{P} \left(\tilde{x}^4 + 4\eta\delta\tilde{x}^3 + \cancel{6\eta^2\delta^2\tilde{x}^2} + \right. \\ &\quad \left. + 4\eta^3\delta^3\tilde{x} - 6\tilde{x}^2\tilde{y}^2 - 18\eta\delta\tilde{x}\tilde{y}^2 - \right. \\ &\quad \left. - 6\eta^2\delta^2\tilde{y}^2 + \tilde{y}^4 \right) \quad (2a)\end{aligned}$$

$$\begin{aligned}\ddot{\tilde{y}} + k_y \tilde{y} &= -4 \frac{b_4}{P} \left(\tilde{x}^3\tilde{y} + 3\eta\delta\tilde{x}^2\tilde{y} + 3\eta^2\delta^2\tilde{x}\tilde{y}^2 + \right. \\ &\quad \left. + \eta^3\delta^3\tilde{y} - \tilde{x}\tilde{y}^3 - \eta\delta\tilde{y}^3 \right) \quad (2b)\end{aligned}$$

These time-derivatives are

$$\begin{aligned}\Delta Q_x &= -\frac{1}{4\pi P} \oint_{H_0} \beta_v b_4 \left(\tilde{x}^3 + 4\eta\delta\tilde{x}^2 + \cancel{6\eta^2\delta^2\tilde{x}} + 4\eta^3\delta^3 + \right. \\ &\quad \left. - 6\tilde{x}\tilde{y}^2 - 12\eta\delta\tilde{y}^2 \right) ds \quad (3a)\end{aligned}$$

where we have ignored the last two terms at the r.h.s. of eq. 2a since they do not give contribution to first-order

$$\begin{aligned}\Delta Q_y &= \frac{1}{4\pi P} \oint q \beta_v b_4 \left(\tilde{x}^3 + 3\eta\delta\tilde{x}^2 + 3\eta^2\delta^2\tilde{x} + \eta^3\delta^3 + \right. \\ &\quad \left. - \tilde{x}\tilde{y}^2 - \eta\delta\tilde{y}^2 \right) ds \quad (3b)\end{aligned}$$

where the integrals are taken over the full circumference of the ring -

To first-order, the time-shifts are averaged over several libration oscillations thus the terms in \tilde{x}^4 and \tilde{x}^2 in eq.(3a)⁽¹⁾ give in average zero contribution. ~~Similarly~~

Again in first-order, we can set

$$\tilde{x}^2 = \frac{\varepsilon_H}{\pi} \beta_H \cos^2 \psi_H$$

$$\tilde{y}^2 = \frac{\varepsilon_V}{\pi} \beta_V \cos^2 \psi_V$$

~~Also~~ Finally

$$\Delta Q_H = - \frac{\delta^3}{\pi p} \int b_4 \beta_H \gamma^3 ds +$$

$$- \frac{\delta \varepsilon_H}{\pi p} \int b_4 \beta_H \gamma^2 \cos^2 \psi_H ds +$$

(4a)

$$+ \frac{3 \delta \varepsilon_V}{\pi p} \int b_4 \beta_H \beta_V \gamma \cos^2 \psi_V ds$$

and

(4)

$$\Delta Q_V = \frac{\delta^3}{\pi p} \int b_4 \beta_V \eta^3 ds +$$

$$- \frac{3\delta\varepsilon_V}{\pi p} \int b_4 \beta_V^2 \eta \cos^2 \varphi_V ds +$$

$$+ \frac{3\delta\varepsilon_H}{\pi p} \int b_4 \beta_V \beta_H \eta \cos^2 \varphi_H ds \quad (4b)$$

The term in δ^3 has an equivalent in the second-order contribution from the sextupoles. By properly arranging the sextupoles in families, this term should be compensated for by the sextupole strength alone (an exercise to be done, though). We will neglect here for the term in δ^3 in both eqs (4a and b). For the remaining terms, it is sufficient to take

$$\langle \cos^2 \varphi_V \rangle = \langle \cos^2 \varphi_H \rangle = 1/2$$

and

$$\Delta Q_H = - \frac{\delta\varepsilon_H}{2\pi p} \int b_4 \beta_H^2 \eta ds + \frac{3\delta\varepsilon_V}{2\pi p} \int b_4 \beta_H \beta_V \eta ds \quad (5a)$$

(5)

$$\Delta Q_V = - \frac{\delta \varepsilon_V}{2\pi\rho} \int b_4 \beta_V^2 \eta \, ds + \frac{3\delta \varepsilon_H}{2\pi\rho} \int b_4 \beta_V \beta_H \eta \, ds \quad (5)$$

A. Contribution from Dipole error (systematic)

$$b_4 = -4.7 \times 10^{-4} / \text{in}^4$$

$$\Delta Q_H = -b_4 \langle \eta \beta_H^2 \rangle \delta \varepsilon_H + 3b_4 \langle \eta \beta_H \beta_V \rangle \delta \varepsilon_V \quad (6a)$$

$$\Delta Q_V = 3b_4 \langle \eta \beta_H \beta_V \rangle \delta \varepsilon_H - b_4 \langle \eta \beta_V^2 \rangle \delta \varepsilon_V \quad (6b)$$

where $\langle \dots \rangle$ is the average value over a single dipole

$$\langle \eta \beta_H^2 \rangle = 862.1 \text{ m}^3$$

$$\langle \eta \beta_V^2 \rangle = 682.6 \text{ m}^3$$

$$\langle \eta \beta_H \beta_V \rangle = 505.3 \text{ m}^3$$

(6)

	β_H	β_V	γ
first end } dipole	38.3 m	11.8 m	1.31 m
middle	23.0	23.0	1.07
second end }	11.1	40.3	0.80
half ellipse QF	45.0	10.0	1.45
half ellipse QD	10.0	45.0	0.80

Beam parameters from RHIC CD

Gold - 100 GeV

After 10 hours with 1.1×10^9

$$\epsilon_n = 30 \pi \text{ mm.mrad} \quad (95\% \text{ of beam, H and V})$$

For a $60_{H,V}$ good-field criterion

$$\epsilon_H = \epsilon_V = 1.8 \pi \text{ mm.mrad}$$

Also the rf-bucket size with design rf system

$$\delta = \pm 0.27\%$$

$\delta \varepsilon_H$ $\delta \varepsilon_V$ ΔQ_H

- 0.0158

0.0278

 ΔQ_V

0.0278

- 0.0125

There is clearly a need for correction ▷

Two families (1) before Q_F (2) before Q_D

Two-Sight Contribution from Correctors

$$\Delta Q_H = -\frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{4D} \langle \beta_H^2 \eta \rangle_D \right) \delta \varepsilon_H +$$

$$+ \frac{3ML}{2\pi\rho} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) \delta \varepsilon_V \quad (7a)$$

$$\Delta Q_D = \frac{3ML}{2\pi\rho} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) \delta \varepsilon_H +$$

$$- \frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_V^2 \eta \rangle_F + b_{4D} \langle \beta_V^2 \eta \rangle_D \right) \delta \varepsilon_V \quad (7b)$$

(8)

There are four terms to be cancelled with only two parameters (b_{4F} and b_{4D}) - For ΔQ_H

$$-\frac{ML}{2\pi P} \left(b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{4D} \langle \beta_H^2 \eta \rangle_D \right) = b_4 \langle \eta \beta_H^2 \rangle \quad (8)$$

$$\frac{3ML}{2\pi P} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) = -3b_4 \langle \eta \beta_H \beta_V \rangle \quad (9)$$

and for ΔQ_V

$$\frac{3ML}{2\pi P} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) = -3b_4 \langle \eta \beta_H \beta_V \rangle \quad (10)$$

$$-\frac{ML}{2\pi P} \left(b_{4F} \langle \beta_V^2 \eta \rangle_F + b_{4D} \langle \beta_V^2 \eta \rangle_D \right) = b_4 \langle \eta \beta_V^2 \rangle \quad (11)$$

M number of correctors per family
L length of each corrector

Observe that condition (9) and (10) are identical -
 Therefore there are really only three conditions to be satisfied with only two parameters
 choose only (8) and (9)

$$\langle \beta_H^2 \gamma \rangle_F = 2936 \text{ m}^3$$

$$\langle \beta_H^2 \gamma \rangle_D = 80.$$

$$\langle \beta_H \beta_V \gamma \rangle_F = 652.5$$

$$\langle \beta_H \beta_V \gamma \rangle_D = 360.$$

$$\langle \beta_V^2 \gamma \rangle_F = 145.$$

$$\langle \beta_V^2 \gamma \rangle_D = 1620$$

$$\left\{ \begin{array}{l} 2936 \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + \rho_0 \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = - \cancel{852.1} \\ 652.5 \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + 360 \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = - 505.3 \end{array} \right.$$

$$M = 144$$

$$L = 0.5 \text{ m}$$

$$\rho = 240 \text{ m}$$

The required corrector strengths are

$$\left\{ \begin{array}{l} b_{4F} = -5.7 b_4 = -26.8 \times 10^{-4} / \text{in}^4 \\ b_{4D} = -19.5 b_4 = -91.7 \times 10^{-4} / \text{in}^4 \end{array} \right.$$

There is though a residual tune shift given by

$$\begin{aligned} \Delta Q_D &= -\frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_v^2 \eta \rangle_F + b_{4D} \langle \beta_v^2 \eta \rangle_D \right) \delta \varepsilon_v \\ &\quad + \text{dipole contribution } (-0.0125) \\ &= 0.085 \quad (\text{too large?}) \end{aligned}$$

Instead of (8) and (9) make use of (10) and (11)

$$\left\{ \begin{array}{l} 652.5 \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + 360 \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = -505.7 \\ 145 \cdot \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{4} \right) + 1820 \cdot \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = -682.6 \end{array} \right.$$

From here the required corrector strengths are

$$\begin{cases} b_{4F} = -12.1 \quad b_4 = -56.9 \times 10^{-4} / \text{in}^4 \\ b_{4D} = -7.9 \quad b_4 = -37.1 \times 10^{-4} / \text{in}^4 \end{cases}$$

Total residual now is

$$\Delta Q_H = -\frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{4D} \langle \beta_H^2 \eta \rangle_D \right) \eta \epsilon_H$$

+ dipole contribution (-0.058)

$$= 0.0155 \quad (\text{too large})$$

It seems the only solution is to
shim the dipoles to eliminate the b_4^+ error.
Unless one wants to make use of correctors
also in the insertions as a third family -